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Mathematics
Higher level
Paper 3 – sets, relations and groups

Thursday 21 November 2019 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Let $A = \{1, 3, 4, 5, 8, 9\}$, $B = \{1, 5, 6, 7, 9\}$ and $C = \{1, 2, 7, 8, 9\}$.

- (a) (i) Find $(A \setminus B) \setminus C$ where \setminus represents set difference.
 - (ii) Find $A \setminus (B \setminus C)$.
 - (iii) Hence determine whether set difference is associative. [5]
- (b) Find $(A \Delta B) \Delta C$ where Δ represents symmetric difference. [2]
- (c) By considering the sets A , B and C , determine whether symmetric difference is distributive over intersection. [5]

2. [Maximum mark: 14]

The set $\{-4, -3, -2, -1, 0, 1, 2, 3\}$ together with the binary operation, $*$, forms a group, as defined in the following Cayley table.

| * | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|----|----|----------|----|----|----|----|----------|----|
| -4 | 0 | 1 | 2 | 3 | -4 | -3 | -2 | -1 |
| -3 | 1 | <i>a</i> | 3 | -4 | -3 | -2 | -1 | 0 |
| -2 | 2 | 3 | -4 | -3 | -2 | -1 | 0 | 1 |
| -1 | 3 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| 0 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| 1 | -3 | <i>b</i> | -1 | 0 | 1 | 2 | <i>c</i> | -4 |
| 2 | -2 | -1 | 0 | 1 | 2 | 3 | -4 | -3 |
| 3 | -1 | 0 | 1 | 2 | 3 | -4 | -3 | -2 |

- (a) (i) Explain what is meant by the term Latin square.
 - (ii) Hence write down the values of a , b and c . [4]
- (b) (i) Write down the identity element of this group.
 - (ii) Hence state the inverse of the element -4 . [2]
- (c) By finding the order of elements, determine whether this group is cyclic. [3]
- (d) Find a subgroup of order 4. [2]

(This question continues on the following page)

(Question 2 continued)

There is an isomorphism, f , from the group $\{-4, -3, -2, -1, 0, 1, 2, 3\}, *$ to the group $\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8$ where $+_8$ is the operation addition modulo 8.

(e) Given that $f(1) = 1$, find the value of $f(-3)$. [3]

3. [Maximum mark: 13]

(a) Let V be the set of three-dimensional vectors. A relation R is defined on V by aRb if and only if $a \cdot b = 0$. Determine with reasons whether R is

- (i) reflexive;
- (ii) symmetric;
- (iii) transitive. [3]

(b) Let W be the set of **non-zero** three-dimensional vectors. A relation S is defined on W by aSb if and only if $a \times b = \mathbf{0}$. Determine with reasons whether S is

- (i) reflexive;
- (ii) symmetric;
- (iii) transitive. [5]

(c) (i) Exactly one of R and S is an equivalence relation. State which relation this is.

(ii) For this equivalence relation, $\begin{pmatrix} -2 \\ y \\ -4 \end{pmatrix}$ belongs to the equivalence class

containing $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. Find the value of y . [3]

(d) The relation S from part (b) is now defined on the set V from part (a). Determine, with a reason, whether S is transitive on V . [2]

Turn over

4. [Maximum mark: 11]

- (a) Let $\{G, *\}$ be a group.
Prove that $\{G, *\}$ has **exactly** one identity element. [3]
- (b) The binary operation \otimes is defined on the set of real numbers by $a \otimes b = a|b|$.
- (i) Determine whether \otimes is associative, justifying your answer.
- (ii) Determine whether there is an identity element for \otimes , justifying your answer. [8]
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